

NAG Toolbox for MATLAB

f08qf

1 Purpose

f08qf reorders the Schur factorization of a real general matrix.

2 Syntax

```
[t, q, ifst, ilst, info] = f08qf(compq, t, q, ifst, ilst, 'n', n)
```

3 Description

f08qf reorders the Schur factorization of a real general matrix $A = QTQ^T$, so that the diagonal element or block of T with row index **ifst** is moved to row **ilst**.

The reordered Schur form \tilde{T} is computed by an orthogonal similarity transformation: $\tilde{T} = Z^T T Z$. Optionally the updated matrix \tilde{Q} of Schur vectors is computed as $\tilde{Q} = QZ$, giving $A = \tilde{Q}\tilde{T}\tilde{Q}^T$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **compq** – string

Indicates whether the matrix Q of Schur vectors is to be updated.

compq = 'V'

The matrix Q of Schur vectors is updated.

compq = 'N'

No Schur vectors are updated.

Constraint: **compq** = 'V' or 'N'.

2: **t(ldt,*)** – double array

The first dimension of the array **t** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

The n by n upper quasi-triangular matrix T in canonical Schur form, as returned by f08pe.

3: **q(ldq,*)** – double array

The first dimension, **ldq**, of the array **q** must satisfy

if **compq** = 'V', **ldq** $\geq \max(1, n)$;

if **compq** = 'N', **ldq** ≥ 1 .

The second dimension of the array must be at least $\max(1, n)$ if **compq** = 'V' and at least 1 if **compq** = 'N'

If **compq** = 'V', **q** must contain the n by n orthogonal matrix Q of Schur vectors.

4: **ifst** – int32 scalar

5: **ilst** – int32 scalar

ifst and **ilst** must specify the reordering of the diagonal elements or blocks of T . The element or block with row index **ifst** is moved to row **ilst** by a sequence of exchanges between adjacent elements or blocks.

Constraint: $1 \leq \text{ifst} \leq n$ and $1 \leq \text{ilst} \leq n$.

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **t**.

n , the order of the matrix T .

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldt, **ldq**, **work**

5.4 Output Parameters

1: **t(ldt,*)** – double array

The first dimension of the array **t** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

t contains the updated matrix \tilde{T} . See also Section 8.

2: **q(ldq,*)** – double array

The first dimension, **ldq**, of the array **q** must satisfy

if **compq** = 'V', $\text{ldq} \geq \max(1, n)$;
if **compq** = 'N', $\text{ldq} \geq 1$.

The second dimension of the array must be at least $\max(1, n)$ if **compq** = 'V' and at least 1 if **compq** = 'N'

If **compq** = 'V', **q** contains the updated matrix of Schur vectors.

If **compq** = 'N', **q** is not referenced.

3: **ifst** – int32 scalar

4: **ilst** – int32 scalar

If **ifst** pointed to the second row of a 2 by 2 block on entry, it is changed to point to the first row. **ilst** always points to the first row of the block in its final position (which may differ from its input value by ± 1).

5: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **compq**, 2: **n**, 3: **t**, 4: **ldt**, 5: **q**, 6: **ldq**, 7: **ifst**, 8: **ilst**, 9: **work**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info = 1

Two adjacent diagonal elements or blocks could not be successfully exchanged. This error can only occur if the exchange involves at least one 2 by 2 block; it implies that the problem is very ill-conditioned, and that the eigenvalues of the two blocks are very close. On exit, T may have been partially reordered, and **ilst** points to the first row of the current position of the block being moved; Q (if requested) is updated consistently with T .

7 Accuracy

The computed matrix \tilde{T} is exactly similar to a matrix $(T + E)$, where

$$\|E\|_2 = O(\epsilon)\|T\|_2,$$

and ϵ is the *machine precision*.

Note that if a 2 by 2 diagonal block is involved in the reordering, its off-diagonal elements are in general changed; the diagonal elements and the eigenvalues of the block are unchanged unless the block is sufficiently ill-conditioned, in which case they may be noticeably altered. It is possible for a 2 by 2 block to break into two 1 by 1 blocks, that is, for a pair of complex eigenvalues to become purely real. The values of real eigenvalues however are never changed by the reordering.

8 Further Comments

The total number of floating-point operations is approximately $6nr$ if **compq** = 'N', and $12nr$ if **compq** = 'V', where $r = |\mathbf{ifst} - \mathbf{ilst}|$.

The input matrix T must be in canonical Schur form, as is the output matrix \tilde{T} . This has the following structure.

If all the computed eigenvalues are real, T is upper triangular and its diagonal elements are the eigenvalues.

If some of the computed eigenvalues form complex conjugate pairs, then T has 2 by 2 diagonal blocks. Each diagonal block has the form

$$\begin{pmatrix} t_{ii} & t_{i,i+1} \\ t_{i+1,i} & t_{i+1,i+1} \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \alpha \end{pmatrix}$$

where $\beta\gamma < 0$. The corresponding eigenvalues are $\alpha \pm \sqrt{\beta\gamma}$.

The complex analogue of this function is f08qt.

9 Example

```
compq = 'No update';
t = [0.8, -0.11, 0.01, 0.03;
     0, -0.1, 0.25, 0.35;
     0, -0.65, -0.1, 0.2;
     0, 0, 0, -0.1];
q = [0];
ifst = int32(2);
ilst = int32(1);
[tOut, qOut, ifstOut, ilstOut, info] = f08qf(compq, t, q, ifst, ilst)

tOut =
    -0.1000    -0.6463     0.0874     0.2010
     0.2514    -0.1000     0.0927     0.3505
```

	0	0	0.8000	-0.0117
	0	0	0	-0.1000
qOut =				
0				
ifstOut =				
	2			
ilstOut =				
	1			
info =				
	0			
